

Reply to Comment on “ Universal Fluctuations in Correlated Systems”

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Bramwell et al. reply

Watkins Chapman and Rowlands claim to show that the deviation from the FTG distribution, due to the correlations between random variables introduced in the extremal statistics problem in [1] is the result of the slow convergence of the PDF for extreme values, with system size, for effective Gaussian variables. We think this result is correct, but it must be put in the context of recent developments in the field.

Our original motivation for studying correlated extremal statistics explicitly excludes the kind of slow relaxation towards an asymptotic, or thermodynamic limit function discussed in [2] and [3]: The PDF for order parameter fluctuations in the low temperature phase of the 2D-XY model (BHP) is a thermodynamic limit function that is different to the FTG distribution. The model studied is diagonalisable into statistically independent variables, for which the dispersion in amplitudes diverges with system size. The PDF for the extreme (largest) value of these variables is the FTG distribution not the BHP distribution [4]. Conclusion: simple extremal statistics do not explain the observed results. If extremal statistics are relevant then they must apply to more complex (correlated) many body objects, rather than the statistically independent variables of the problem. If this is not the case then extremal statistics are irrelevant for at least this model correlated system.

Our first attempt to look at the extremal statistics of correlated random variables was the model presented in [1] and discussed in the Comment. The authors are probably correct to conclude that the main effect comes from finite size corrections, rather than the correlations introduced in the model. This can therefore be classified as “weak correlation” and is the extreme value equivalent of introducing a finite correlation length in a thermodynamic system and then taking the thermodynamic limit. In a strong correlation limit one would expect deviations from FTG to remain on taking the limit $N \rightarrow \infty$. This scenario has been seen in detail in reference [5], where the extreme value for avalanche sizes in the Sneppen depinning model is seen to follow the BHP distribution over a large range of time and length scales. We also note that renormalization group analysis of extreme value statistics for long range correlated signals shows that the tail of the resulting distribution renormalizes from the $\exp(-y)$ asymptote of the FTG distribution to $y \exp(-y)$ [6], in agreement with the exact asymptotic form for the BHP function [2]. The results of [5] and [6] go somewhere towards confirming our hypothesis, first proposed in [1], that deviations to the FTG distribution introduced by correlations could provide the desired link between extremal statistics and the fluctuations of a global quantity in such correlated systems. We suggest that the results presented here (and therefore in [1]) and in reference [3] pass asymptotically close to this correlated regime.

Finally we remark that, even in the case where global fluctuations *are* described by the FTG distribution, as in ref [7] (see also ref. [8]), any connection with extremal statistics remains unproven. The global quantity is a sum over a macroscopic number of elements, while extremal statistics would require the selection of the biggest element by a “Maxwell’s demon”. The connection between these processes is an open problem. The authors of this comment have presented arguments based on slow relaxation towards the FTG distribution for Gaussian variables [3] which could be a step in the right direction. This is a useful contribution but the arguments presented here and in the references below, show that it is far from the complete picture and that it would be quite wrong to infer that deviation from the FTG distribution in the ensemble of systems discussed is generically due to finite size corrections to the thermodynamic limit. More work needs to be done.

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